

# Eton College King's Scholarship Examination 2020

## MATHEMATICS A

(One and a half hours)

*Candidate Number:*.....

***Please write your candidate number on EVERY sheet.***

***Please answer on the paper in the spaces provided.***

*This paper is divided into two sections:*

*Section I (Short-answer questions) – 50 marks available*

*Section II (Extended questions) – 50 marks available*

*Answer all of Section I and as many questions as you can from Section II.*

*The marks for each part of each question are given in square brackets.*

*Show all your working.*

*No diagram is drawn to scale.*

*If you run out of space, please use the Additional Working space on page 16.*

***Neither calculators nor protractors may be used.***

***ADDITIONAL MATERIALS:      NONE***

**Do not turn over until told to do so.**

## Section I: Short-answer questions (50 marks)

1. Find the value of the following, giving your answers as reduced fractions (mixed, where appropriate):

a)  $1\frac{5}{17} \times 1\frac{1}{33}$  [3]

b)  $1\frac{2}{11} \div 4\frac{1}{3}$  [3]

c)  $1001\frac{1}{5} - 999\frac{4}{5}$  [3]

[Total for Question: 9]

2. Find the value of the following, giving your answer as a decimal or whole number as appropriate:

a)  $1.9 \times 0.006$  [3]

b)  $0.0084 \div 0.0000028$  [3]

c)  $(0.03)^4$  [3]

[Total for Question: 9]

3. Given that  $304050 \times 55 = 16722750$ , find the value of

a)  $167227.5 \div 304.05$  [2]

b)  $1.672275 \div 0.0055$  [2]

[Total for Question: 4]

4. Solve the following equation:

$$3(x - 4) = 5(x - 6) - 4(x - 5)$$

[3]

[Total for Question: 3]

5. Solve the following inequality, leaving  $x$  on the left-hand side in your final answer:

$$-7x - 6 \geq 3(x + 2) - 4$$

[3]

[Total for Question: 3]

6. Solve the following pair of simultaneous equations:

$$5x = 7y + 27$$

$$7x = 5y + 57$$

[4]

[Total for Question: 4]

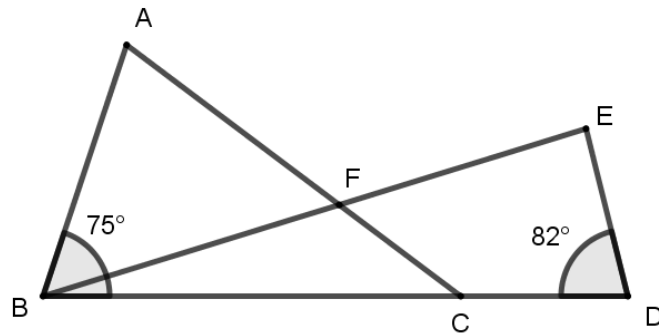
7. The mean of 3.2, 8.1 and  $x$  is one more than the mean of 3.9, 6.2, 6.9 and  $x$ . Calculate  $x$ . [4]

[Total for Question: 4]

8. A survey asked respondents whether they subscribe to online streaming services *Webflix* and *Thames Prime*.  $\frac{49}{120}$  of the respondents subscribe to neither service, and  $\frac{11}{24}$  of the respondents subscribe to Thames Prime. If  $\frac{4}{15}$  of the respondents subscribe to Webflix, what percentage of respondents subscribe to Thames Prime *only*? [5]

[Total for Question: 5]

9. In the following diagram, length  $AC = BC$  and length  $EB = DB$ . Angle  $EDB = 82^\circ$  and angle  $ABC = 75^\circ$ . Calculate angle  $AFE$ . [5]



[Total for Question: 5]

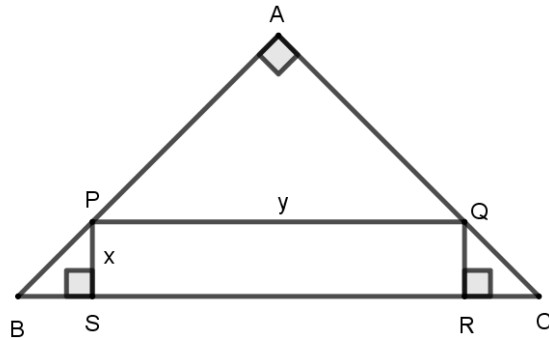
10. In order to number the pages of a book, a printer uses a total of 1017 digits. How many numbered pages are there in the book? [4]

[Total for Question: 4]



**Section II: Extended-answer questions (50 marks)**

11. Right-angled triangle ABC is isosceles. PQRS is a rectangle.  $PS = x$  and  $PQ = y$ .



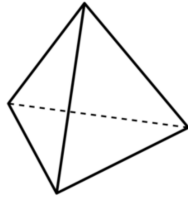
a) Find the area of triangle BPS in terms of  $x$ , justifying your answer. [2]

b) Find the area of triangle APQ in terms of  $y$ , justifying your answer. [4]

c) If triangle ABC has area 225 units<sup>2</sup>, and if  $y = 4x$ , find  $x$ . [4]

[Total for Question: 10]

12. The diagram below shows a *tetrahedron*, which is a three-dimensional solid formed from four equilateral triangular faces:

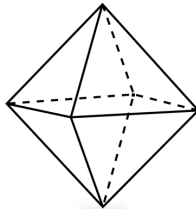


Each *face* is bounded by three *edges*, and three *edges* meet at a single point called a *vertex*.

a) How many edges does a tetrahedron have? [1]

b) How many vertices and edges does a cube have? [2]

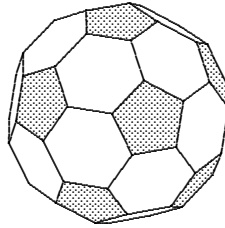
The diagram below shows an *octahedron*.



c) How many vertices and edges does an octahedron have? [2]

*Question 12 continued*

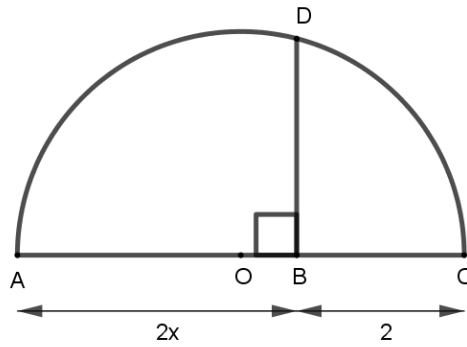
The diagram below shows a *truncated icosahedron*, which has 12 pentagonal faces and 20 hexagonal faces.



- d) How many vertices and edges does a truncated icosahedron have? [5]

[Total for Question: 10]

13. The diagram shows a semicircle, centre O.  $AB = 2x$  and  $BC = 2$ . Angle ABD is a right angle.



a) Find a simplified expression, in terms of  $x$ , for the radius OD. [2]

b) Find a simplified expression, in terms of  $x$ , for the length OB. [2]

c) Using Pythagoras' Theorem, or otherwise, find and simplify an expression in terms of  $x$  for  $BD^2$  (the square of length BD). [3]

d) If the circular arc ADC has length  $122\pi$ , what is length BD? [3]

[Total for Question: 10]

14. Claire collects stamps. Her collection contains UK stamps and non-UK stamps. The ratio of UK to non-UK stamps in her collection is 6:11. Let  $m$  be the number of UK stamps in her collection and  $n$  be the number of non-UK stamps.

a) Find an expression for  $n$  in terms of  $m$ . [1]

If Claire were to swap her best 3 UK stamps for 14 additional non-UK stamps from another collector, the ratio of UK to non-UK stamps in her collection would become 3:7.

b) Form a second equation for  $n$  in terms of  $m$ . [4]

c) By solving the equations in a) and b) simultaneously, work out how many more non-UK than UK stamps Claire had in her original collection. [5]

[Total for Question: 10]

15. Alpha and Beta live at the opposite ends of the same street, 243 metres apart. One day, Alpha had to deliver a parcel to Beta's home, Beta one to Alpha's home. They started their journeys at the same moment, each walked at constant speed, and they returned home immediately after leaving the parcels at their respective destinations.

If Alpha walked at  $1\frac{1}{5}$  m/s and Beta walked at  $1\frac{1}{2}$  m/s:

a) how far from Alpha's home did they first meet? [3]

b) how many seconds after the time Alpha made the parcel delivery at Beta's home did the two meet again? [3]

*Question 15 continued*

Gamma and Delta also live at opposite ends of a different street and need to make reciprocal parcel deliveries in a similar way to Alpha and Beta. They started their journeys at the same moment and each walked at constant speed, but not necessarily the same speeds as Alpha and Beta. They first meet  $c$  metres from Gamma's home and they meet for a second time  $(1000 - 2c)$  metres from Delta's home. *You may assume they meet for the second time after both have delivered their parcels.*

- c) Find how long their street is, leaving your answer in terms of  $c$ . [4]

[Total for Question: 10]

END OF PAPER

ADDITIONAL WORKING