UK Maths Trust

Senior Mathematical Challenge

Tuesday 3 October 2023

Organised by the United Kingdom Mathematics Trust

supported by Overleaf

Candidates must be full-time students at secondary school or FE college.
England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **90 minutes**. No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank paper for rough working is allowed; **squared paper, calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil**. Mark at most one of the options, A, B, C, D, or E, on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed**. The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules**: All candidates start with 25 marks; 0 marks are awarded for each question left unanswered; 4 marks are awarded for each correct answer; 1 mark is deducted for each incorrect answer (to discourage guessing).
7. **Your Answer Sheet will be read by a machine**. Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, doodle, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.
8. **The questions on this paper are designed to challenge you to think, not to guess**. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.
9. To accommodate candidates sitting at other times, please do not discuss the paper on the internet until **08:00 BST on Thursday 5 October**.

Enquiries about the Senior Mathematical Challenge should be sent to:

challenges@ukmt.org.uk

www.ukmt.org.uk

1. What is the value of $\sqrt{\dfrac{2023}{2+0+2+3}}$?

A 13 B 15 C 17 D 19 E 21

2. What is the difference between one-third and 0.333?

A 0 B $\dfrac{3}{1000}$ C $\dfrac{1}{3000}$ D $\dfrac{3}{10000}$ E $\dfrac{1}{30000}$

3. The base of a triangle is increased by 20% and its height is decreased by 15%.

What happens to its area?

A It decreases by 3% B It remains the same C It decreases by 2%
D It increases by 2% E It increases by 5%

4. In 2016, the world record for completing a 5000m three-legged race was 19 minutes and 6 seconds. It was set by Damian Thacker and Luke Symonds in Sheffield.

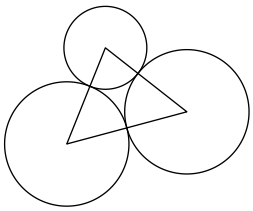
What was their approximate average speed in km/h?

A 10 B 12 C 15 D 18 E 25

5. Three circles with radii 2, 3 and 3 touch each other, as shown in the diagram.

What is the area of the triangle formed by joining the centres of these circles?

A 10 B 12 C 14 D 16 E 18



6. How many lines of three adjacent cells can be chosen from this grid, horizontally, vertically or diagonally, such that the sum of the numbers in the three cells is a multiple of three?

A 30 B 24 C 18 D 12 E 6

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

7. A sequence begins 2023, 2022, 1, ... After the first two terms, each term is the positive difference between the previous two terms.

What is the value of the 25th term?

A 2010 B 2009 C 2008 D 2007 E 2006

8. What is the value of $99(0.\dot{4}\dot{9} - 0.\dot{4})$?

A 5 B 4 C 3 D 2 E 1

9. When completed correctly, the cross number is filled with four three-digit numbers.

What digit is *?

A 0 B 1 C 2
D 4 E 6

Across
1. A square
3. A fourth power

Down
1. Twice a fifth power
2. A cube

1	*	2
3		

10. How many of the numbers 6, 7, 8, 9, 10 are factors of the sum $2^{2024} + 2^{2023} + 2^{2022}$?

A 1 B 2 C 3 D 4 E 5

11. Wenlu, Xander, Yasser and Zoe make the following statements:

Wenlu: "Xander is lying." Xander: " Yasser is lying."

Yasser: "Zoe is telling the truth." Zoe: "Wenlu is telling the truth."

What are the possible numbers of people telling the truth?

A 1 or 2 B 1 or 3 C 2 D 2 or 3 E 3

12. The greatest power of 7 which is a factor of 50! is 7^k $(n! = 1 \times 2 \times 3 \times 4 \times \ldots \times (n-1) \times n)$.

What is k?

A 4 B 5 C 6 D 7 E 8

13. $PQRST$ is a regular pentagon. The point U lies on ST such that $\angle QPU$ is a right angle.

What is the ratio of the interior angles in triangle PUT?

A 1 : 3 : 6 B 1 : 2 : 4 C 2 : 3 : 4 D 1 : 4 : 8 E 1 : 3 : 5

14. The points $P\ (d, -d)$ and $Q\ (12 - d, 2d - 6)$ both lie on the circumference of the same circle whose centre is the origin.

What is the sum of the two possible values of d?

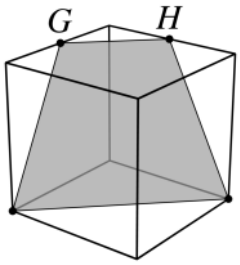
A -16 B -4 C 4 D 8 E 16

15. In Bethany's class of 30 students, twice as many people played basketball as played football. Twice as many played football as played neither.

Which of the following options could have been the number of people who played both?

A 19 B 14 C 9 D 5 E 0

16. G and H are midpoints of two adjacent edges of a cube. A trapezium-shaped cross-section is formed cutting through G, H and two further vertices, as shown. The edge-length of the cube is $2\sqrt{2}$.



What is the area of the trapezium?

A 9 B 8 C $4\sqrt{5}$ D $4\sqrt{3}$ E $4\sqrt{2}$

17. The number $M = 124563987$ is the smallest number which uses all the non-zero digits once each and which has the property that none of the pairs of its consecutive digits makes a prime number. For example, the 5th and 6th digits of M make the number 63 which is not prime. N is the largest number which uses all the non-zero digits once each and which has the property that none of the pairs of its consecutive digits makes a prime number.

What are the 5th and 6th digits of N?

A 6 and 3 B 5 and 4 C 5 and 2 D 4 and 8 E 3 and 5

18. How many solutions are there of the equation $1+2\sin X-4\sin^2 X-8\sin^3 X=0$ with $0^\circ < X < 360^\circ$?

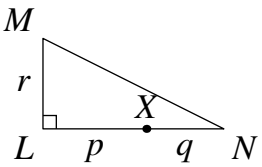
A 1 B 2 C 4 D 6 E 8

19. The expression $\dfrac{7n+12}{2n+3}$ takes integer values for certain integer values of n.

What is the sum of all such integer values of the expression?

A 4 B 8 C 10 D 12 E 14

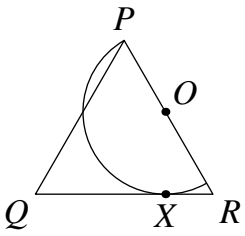
20. Triangle LMN represents a right-angled field with $LM = r$, $LX = p$ and $XN = q$. Jenny and Vicky walk at the same speed in opposite directions along the edge of the field, starting at X at the same time. Their first meeting is at M.



Which of these expressions gives q in terms of p and r?

A $\dfrac{p}{2}+r$ B $\sqrt{p^2+r^2}+\dfrac{p}{2}$ C $\dfrac{pr}{2p+r}$ D $\dfrac{p}{2}$ E 1

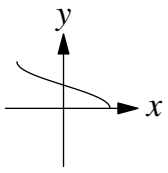
21. Triangle PQR is equilateral. A semicircle with centre O is drawn with its diameter on PR so that one end is at P and the curved edge touches QR at X. The radius of the semicircle is $\sqrt{3}$.

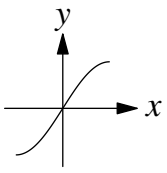


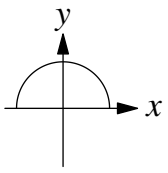
What is the length of QX?

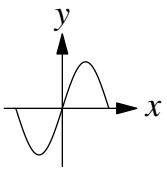
A $\sqrt{3}$ B $2-\sqrt{3}$ C $2\sqrt{3}-1$ D $1+\sqrt{3}$ E $2\sqrt{3}$

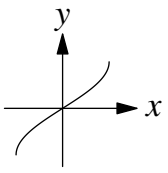
22. Which diagram could be a sketch of the curve $y = \sin(\cos^{-1} x)$?

A 

B 

C 

D 

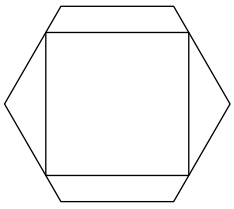
E 

23. The length of a rectangular piece of paper is three times its width. The paper is folded so that one vertex lies on top of the opposite vertex, thus forming a pentagonal shape.

What is the area of the pentagon as a fraction of the area of the original rectangle?

A $\dfrac{2}{3}$ B $\dfrac{11}{16}$ C $\dfrac{12}{17}$ D $\dfrac{13}{18}$ E $\dfrac{14}{19}$

24. A square has its vertices on the edges of a regular hexagon. Two of the edges of the square are parallel to two edges of the hexagon, as shown in the diagram. The sides of the hexagon have length 1.



What is the length of the sides of the square?

A $\dfrac{5}{4}$ B $3-\sqrt{3}$ C $\dfrac{4}{3}$ D $\sqrt{2}$ E $\dfrac{3}{2}$

25. What is the area of the part of the xy-plane within which $x^3y^2 - x^2y^2 - xy^4 + xy^3 \geq 0$ and $0 \leq x \leq y$?

A $\dfrac{1}{4}$ B $\dfrac{1}{2}$ C 1 D 2 E 4

